Discussion of "Reevaluation of Deflection Prediction for Concrete Beams Reinforced with Steel and Fiber Reinforced Polymer Bars" by Peter H. Bischoff

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The author has evaluated some of the approaches commonly used to account for tension stiffening in the calculation of the short-term deflection of reinforced concrete flexural members. The most commonly used approach involves the determination of an average effective moment of inertia (I_e) for a cracked member for use in elastic deflection calculations. Several different empirical equations are available for calculating I_e and these are discussed in considerable depth.

The approach used in North America (ACI 2002; CSA 2004) and elsewhere (SAA 2001) involves the calculation of I_e using the well-known equation developed by Branson (1965)

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leqslant I_g \tag{1}$$

where I_{cr}=moment of inertia of the fully cracked transformed cross section; I_g=moment of inertia of the gross cross section; M_{cr}=cracking moment; and M_a=applied moment at the critical section.

The author correctly points out that Eq. (1) overestimates the average stiffness for reinforced concrete members containing relatively small quantities of steel reinforcement (when $\rho = A_{st}/bd$ is less than about 1%) and, for very lightly reinforced members (where I_g/I_{cr} is large), the use of Eq. (1) grossly underestimates short-term deflections. The situation is even worse for members containing FRP where I_g/I_{cr} is very large indeed. As an improvement, the author has proposed the following equation

$$I_e = \left(\frac{I_{cr}}{1-(1-I_{cr}/I_{unc})(M_{cr}/M_a)^2}\right) \leqslant I_{unc}$$

where I_{unc}=moment of inertia of the uncracked transformed cross section (which is approximately equal to I_g for lightly reinforced members).

To test the applicability of the above equation [Eq. (7) in the original paper] for lightly reinforced members containing steel reinforcement, the discusser has here compared the measured moment versus deflection response with the calculated responses using Eqs. (1) and (7) from the original paper for eight simply supported singly reinforced concrete one-way slabs containing tensile steel quantities in the range $0.0018 < \rho < 0.01$. The slabs (designated S1 to S3, S8, and Z1 to Z4) were all prismatic, of rectangular section (850-mm wide) and contained a single layer of longitudinal tensile steel reinforcement (E_s=200,000 MPa and f_{sy}=500 MPa) at an effective depth d. Slabs S1 to S3 and S8 were simply supported over a span of 3,500 mm and were subjected to a single concentrated load at midspan. The results of these tests have been reported elsewhere (Gilbert and Smith 2004). Slabs Z1 to Z4 each had a span of 2,000 mm and were subjected to two concentrated loads applied at the third span points. Details of each slab are given in Table 1, including relevant geometric and concrete material properties.

Fig. 1 provides a comparison between the measured moment versus instantaneous deflection response at midspan of each slab with the calculated responses obtained using Branson's equation [Eq. (1)] and Bischoff's equation [Eq. (7)]. In all cases, Branson's equation underestimates the postcracking instantaneous deflection of the slab and, for very lightly reinforced slabs, Eq. (1) grossly underestimates deflection. Branson's equation generally provides a better agreement for the more heavily reinforced slabs. In all cases, Bischoff's equation provides a much closer agreement with the measured deflection over the full range of steel reinforcement ratios considered.

Bischoff's equation more accurately models the instantaneous tension stiffening phenomenon than the Branson equation used in ACI 318-02 (2002) and CSA A23.3-04 (2004) and serious consideration should be given to adopting it in the next editions of these standards.

Table 1. Designation and Details of Slab Specimens

Slab	Depth h (mm)	Effect. Depth d (mm)	Steel Area A_{st} (mm^2)	$\rho = A_{st}/bd$	f'_c (MPa)	E_c (MPa)	Tensile Strength f_t (MPa)
S1	110	92	141	0.00180	37.3	26,800	3.39
S2	110	91	227	0.00293	37.3	26,800	3.39
S3	110	90	354	0.00463	37.3	26,800	3.39
S8	110	89	339	0.00448	52.2	30,700	4.16
Z1	100	82	141	0.00202	38.4	27,390	3.60
Z2	100	81	227	0.00330	38.4	27,390	3.60
Z3	100	80	354	0.00521	38.4	27,390	3.60
Z4	100	79	565	0.00841	48.8	30,500	4.04

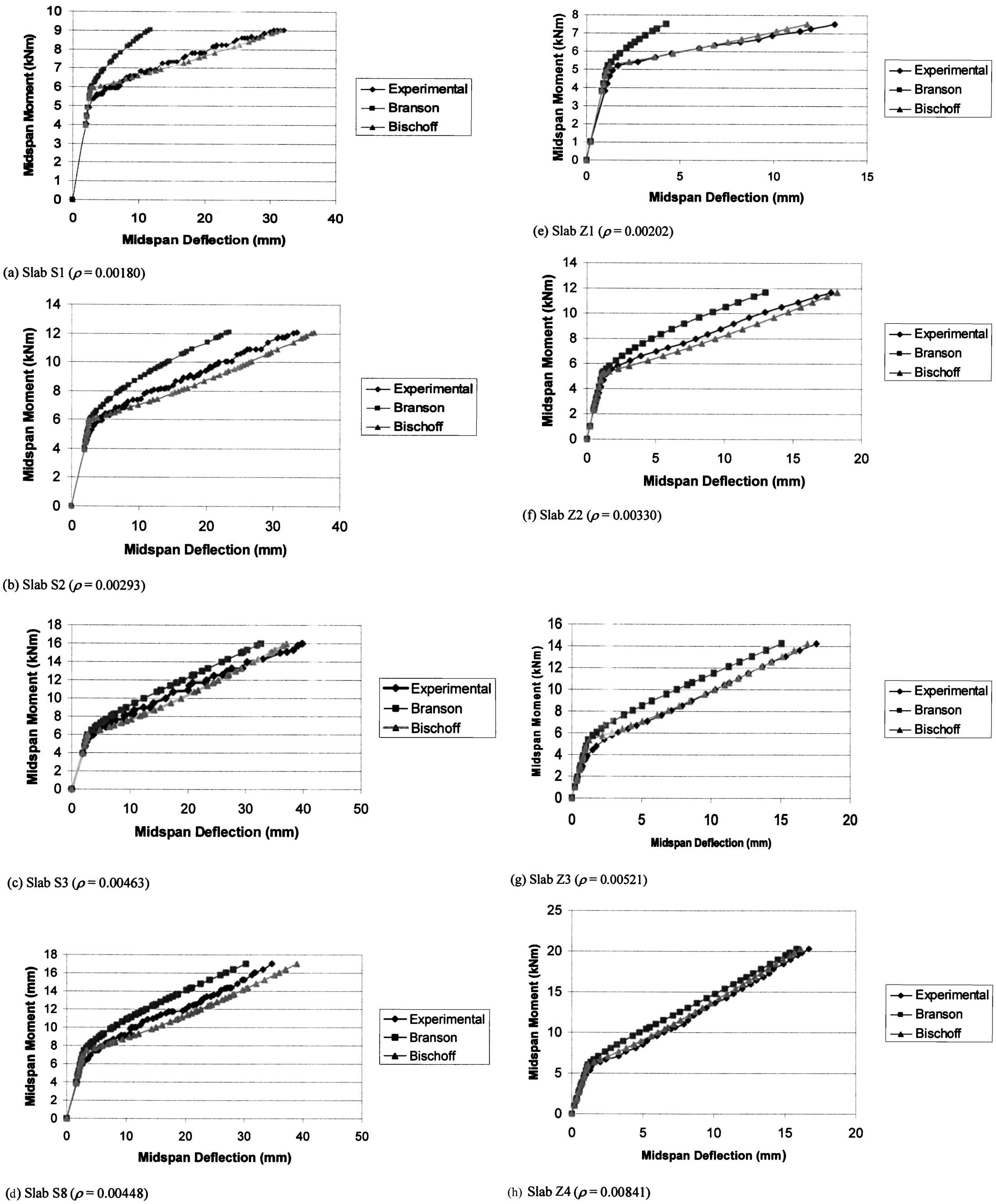

Fig. 1. Midspan moment versus deflection measurements and predictions

Erratum

The following correction should be made to the original paper: Cracked section properties in Appendix IV should read:

$$\frac{I_{cr}}{bd^3} = k_{cr}^3/3 + n\rho(1 - k_{cr})^2$$

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Closure to "Reevaluation of Deflection Prediction for Concrete Beams Reinforced with Steel and Fiber Reinforced Polymer Bars" by Peter H. Bischoff

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The contribution by Professor Gilbert completes the writer's paper by providing a timely comparison of computed beam deflection with experimental results for flexural members containing small amounts of steel reinforcement. Sufficient data exists to assess Bischoff's equation [Eq. (7)] for calculating deflection of FRP reinforced concrete beams, and it is generally accepted that Branson's equation underestimates deformation using this type of reinforcement. However, little information is readily available to evaluate the suitability of using either Branson's expression [Eq. (1) in the discussion] or Bischoff's approach [Eq. (7) of the original paper] for members with low steel reinforcement ratios. The comparison provided by Dr. Gilbert clearly indicates that the proposed form of Eq. (7) correctly accounts for tension stiffening in steel reinforced concrete beams and is further evidence of the need to adopt this alternative approach in our concrete design standards.

It is important to realize that results from this paper imply tension stiffening is independent of both the bar type and reinforcing ratio when this effect is measured relative to the concrete contribution at first cracking. In other words, the same tension stiffening *factor* is used for both steel and FRP reinforced concrete. The tension stiffening component in Branson's equation unfortunately depends on the I_g/I_{cr} ratio, and only works well for flexure members with I_g/I_{cr} less than 3 (corresponding to steel reinforced concrete beams with a reinforcing ratio greater than about 1%). Hence, deflection is underestimated for beams reinforced with FRP bars or with low steel reinforcement ratios. In both cases, the I_g/I_{cr} ratio is much greater than 3. Similarly, Branson's expression does not work well for slender walls with a central layer of reinforcement $(d/h=0.5)$, since the I_g/I_{cr} ratio in this instance can range anywhere from about 15 to 25. Recent concern about the alternative slender wall design procedure adopted by ACI 318-02 (2002) and IBC 2003 (2002) has been expressed by the SEAOSC Slender Wall Task Group (2005), and supports the argument that this procedure significantly underestimates service load deflection of walls when deflection is computed using Branson's equation for the effective moment of inertia I_e.

References

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